

Data Structures & Algorithms



in Python

MICHAEL T. GOODRICH • ROBERTO TAMASSIA • MICHAEL H. GOLDWASSER

Data Structures and Algorithms in Python

Michael T. Goodrich

Department of Computer Science
University of California, Irvine

Roberto Tamassia

Department of Computer Science
Brown University

Michael H. Goldwasser

Department of Mathematics and Computer Science
Saint Louis University

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Deletion

Deleting an item from a binary search tree T is a bit more complex than inserting a new item because the location of the deletion might be anywhere in the tree. (In contrast, insertions are always enacted at the bottom of a path.) To delete an item with key k , we begin by calling `TreeSearch(T, T.root(), k)` to find the position p of T storing an item with key equal to k . If the search is successful, we distinguish between two cases (of increasing difficulty):

- If p has at most one child, the deletion of the node at position p is easily implemented. When introducing update methods for the `LinkedBinaryTree` class in Section 8.3.1, we declared a nonpublic utility, `_delete(p)`, that deletes a node at position p and replaces it with its child (if any), presuming that p has at most one child. That is precisely the desired behavior. It removes the item with key k from the map while maintaining all other ancestor-descendant relationships in the tree, thereby assuring the upkeep of the binary search tree property. (See Figure 11.5.)
- If position p has two children, we cannot simply remove the node from T since this would create a “hole” and two orphaned children. Instead, we proceed as follows (see Figure 11.6):
 - We locate position r containing the item having the greatest key that is strictly less than that of position p , that is, $r = \text{before}(p)$ by the notation of Section 11.1.1. Because p has two children, its predecessor is the rightmost position of the left subtree of p .
 - We use r 's item as a replacement for the one being deleted at position p . Because r has the immediately preceding key in the map, any items in p 's right subtree will have keys greater than r and any other items in p 's left subtree will have keys less than r . Therefore, the binary search tree property is satisfied after the replacement.
 - Having used r 's as a replacement for p , we instead delete the node at position r from the tree. Fortunately, since r was located as the rightmost position in a subtree, r does not have a right child. Therefore, its deletion can be performed using the first (and simpler) approach.

As with searching and insertion, this algorithm for a deletion involves the traversal of a single path downward from the root, possibly moving an item between two positions of this path, and removing a node from that path and promoting its child. Therefore, it executes in time $O(h)$ where h is the height of the tree.

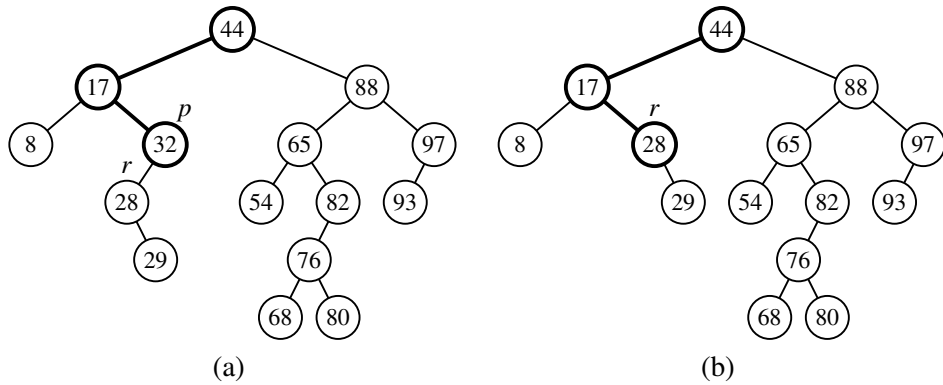


Figure 11.5: Deletion from the binary search tree of Figure 11.4b, where the item to delete (with key 32) is stored at a position p with one child r : (a) before the deletion; (b) after the deletion.

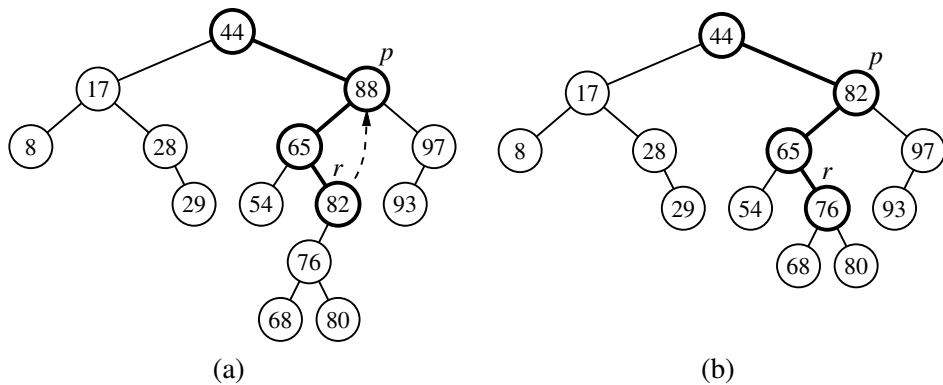


Figure 11.6: Deletion from the binary search tree of Figure 11.5b, where the item to delete (with key 88) is stored at a position p with two children, and replaced by its predecessor r : (a) before the deletion; (b) after the deletion.